MECHANICS (A) UNIT 2   TEST PAPER 10

Take $g = 9.8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. Particles of mass $2m$, $3m$ and $5m$ are placed at the points in the x-y plane with coordinates $(-1, 5)$, $(0, 6)$ and $(3, -2)$ respectively.

   Find the coordinates of the centre of mass of this system of particles.   **(4 marks)**

2. A lorry of mass 3800 kg is pulling a trailer of mass 1200 kg along a straight horizontal road. At a particular moment, the lorry and trailer are moving at a speed of 10 ms^{-1} and accelerating at 0·8 ms^{-2}. The resistances to the motion of the lorry and the trailer are constant and of magnitude 1600 N and 600 N respectively.

   Find the rate, in kW, at which the engine of the lorry is working.   **(4 marks)**

3. A bullet of mass 0·05 kg is fired with speed u ms^{-1} from a gun, which recoils at a speed of $0.008u$ ms^{-1} in the opposite direction to that in which the bullet is fired.

   (a) Find the mass of the gun.   **(2 marks)**

   (b) Find, in terms of u, the kinetic energy given to the bullet and to the gun at the instant of firing.   **(4 marks)**

   (c) If the total kinetic energy created in firing the gun is 5100 J, find the value of u.   **(2 marks)**

4. The acceleration of a particle P at time t s is $\mathbf{a}$ ms^{-2}, where $\mathbf{a} = 4e^{t}\mathbf{i} - e^{t}\mathbf{j}$. When $t = 0$, P has velocity $4\mathbf{i}$ ms^{-1}.

   (a) Find the speed of P when $t = 2$.   **(5 marks)**

   (b) Find the time at which the direction of motion of P is parallel to the vector $5\mathbf{i} - \mathbf{j}$.

       (5 marks)
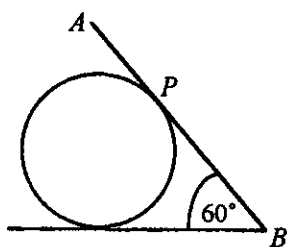
5. A uniform plank AB, of mass 3 kg and length 2 m, rests in equilibrium with the point P in contact with a smooth cylinder. The end B rests on a rough horizontal surface and the coefficient of friction between the plank and the surface is $\frac{1}{3}$. AB makes an angle of 60° with the horizontal.
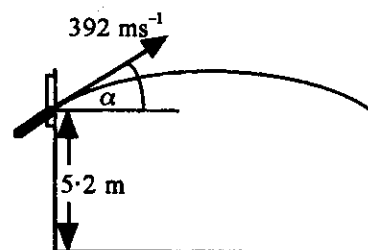
   If the plank is in limiting equilibrium in this position, find

   (a) the magnitude of the force exerted by the cylinder on the plank at P,   **(7 marks)**

   (b) the distance AP.   **(3 marks)**

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6. Two smooth spheres A and B have equal radii and masses 0·4 kg and 0·8 kg respectively. They are moving in opposite directions along the same straight line, with speeds 3 ms^{-1} and 2 ms^{-1} respectively, and collide directly. The coefficient of restitution between A and B is 0·8.

   (a) Calculate the speeds of A and B after the impact, stating in each case whether the direction of motion has been reversed. **(8 marks)**

   (b) Find the kinetic energy, in J, lost in the impact. **(3 marks)**

7. A point of light, P, is moving along a straight line in such a way that, t seconds after passing through a fixed point O on the line, its velocity is v ms^{-1}, where $v = \frac{1}{2}t^2 - 4t + 10$. Calculate

   (a) the velocity of P 6 seconds after it passes O, **(1 mark)**

   (b) the magnitude of the acceleration of P when $t = 1$, **(3 marks)**

   (c) the minimum speed of P, **(3 marks)**

   (d) the distance travelled by P in the fourth second after it passes O. **(5 marks)**

8. A bullet is fired out of a window at a height of 5·2 m above horizontal ground. The initial velocity of the bullet is 392 ms^{-1} at an angle α above the vertical, where $\sin \alpha = \frac{1}{20}$, as shown.
   Find

   (a) the range of times after firing during which the bullet is 15 m or more above ground level,

   (6 marks)

   (b) the greatest height above the ground reached by the bullet, **(3 marks)**

   (c) the horizontal distance travelled by the bullet before it reaches its highest point.

   (3 marks)

   Certain modelling assumptions have been made about the bullet.

   (d) State these assumptions and suggest a way in which the model could be refined.

   (2 marks)

   (e) State, with a reason, whether you think this refinement would make a significant difference to the answers. **(2 marks)**